

4. A. Kudirka, R. Grosh, and P. McFadden, "Heat transfer in two-phase flow of gas-liquid mixtures," *Ind. Chem. Fund.*, 4, No. 3, 339 (1965).
5. A. Kudirka, "Two-phase convective heat transfer," *Chem. Process. Eng.*, 47, No. 11, 43 (1966).
6. T. Ueda, "On upward flow of gas-liquid mixtures in vertical tubes," *Bull. JSME*, 10, No. 42 (1967).
7. J. Collier, *A Survey of Papers on Heat Transfer in Two-Phase Systems* [Russian translation], IL, Moscow (1962).
8. P. M. Krokovnyi, V. E. Nakoryakov, B. G. Pokusaev, and V. A. Utovich, "An electrodiffusion study of horizontal two-phase flow," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 2 (1973).
9. P. M. Krokovnyi, B. G. Pokusaev, and V. A. Utovich, "Measurement of local friction in a two-phase horizontal flow," in: *Hydrodynamics and Heat Transfer* [in Russian], S. S. Kutateladze (editor), ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1972).
10. S. S. Kutateladze, A. P. Burdukov, V. E. Nakoryakov, and V. A. Kuz'min, "Use of an electrochemical method of measuring friction in the hydrodynamics of two-phase media," in: *Proceedings of the Third All-Union Conference on Heat and Mass Transfer* [in Russian], Vol. 2, Naukova Dumka, Kiev (1968).
11. V. A. Malkov and Ya. Ya. Tomsons, "Analysis of circuits for connecting electrodiffusion flow-velocity friction transducers," *Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk*, No. 3, Issue 1 (1978).
12. T. S. Kornienko and M. Kh. Kishinevskii, "A study of mass transfer in turbulent flow in a tube with working parts of different lengths," *Teor. Osn. Khim. Tekh.*, No. 5, 6 (1972).
13. P. V. Shaw and T. J. Hanratty, "Fluctuation in the local rate of turbulent mass transfer to a pipe wall," *AIChE J.*, 10, 475 (1964).
14. T. S. Kornienko, "A study of turbulent mass transfer (heat transfer) at high Prandtl numbers," Author's Abstract of Doctoral Dissertation, Voronezh. Tekhn. Inst., Leningrad (1978).
15. H. Verschoor and S. Stermerding, *Proceedings of a General Discussion on Heat Transfer* (1951).
16. H. G. Schlichting, *Boundary Layer Theory*, McGraw-Hill (1968).
17. S. S. Kutateladze (editor), *Problems of Heat Transfer and Hydraulics of Two Phase Media*, Pergamon (1970).

## CHOICE OF PULSE-FLUIDIZATION CONDITIONS FOR MIXING GRANULATED MATERIALS

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A method is given for determining the best pulse-fluidization conditions for accelerating the mixing of components in a granular material.

These measurements on the mixing kinetics of granular materials in pulse fluidization provide evidence on the rate of the process in relation to the working conditions, i. e., the frequency  $f$ , mark-space ratio  $S$ , and gas flow rate. Visual observations show that rapid mixing corresponds to a definite state of the layer, which may be characterized as the active-piston state. This occurs if the maximum pressure under the bed during the pulse does not exceed  $(1.5-2.5)Mg/S_e$  and causes bed expansion, with only a small amplitude of the oscillation of the upper boundary relative to the mean position and the absence of ejection above the bed. This is termed region A (Fig. 1) in  $S$ - $f$  coordinates. In region B, there are large bubbles, whose escape into the space above the bed is accompanied by considerable ejection of material. In region C, the layer remains largely immobile and very little particle displacement occurs.

However, the mixing rate is not constant within region A for any combination of  $f$  and  $S$ ; there is a definite pair of values for  $f$  and  $S$  (in our case  $f=5$  Hz and  $S=0.5$ ) that produces the most vigorous mixing. This has been observed elsewhere [1], where the effect was described in terms of a natural oscillation frequency of the bed.

The hydrodynamic features of the bed were examined by cinemography in parallel with gas pressure measurement under the distribution grid. Figure 2 shows characteristic pressure oscillograms for  $f$  of 1-5 Hz

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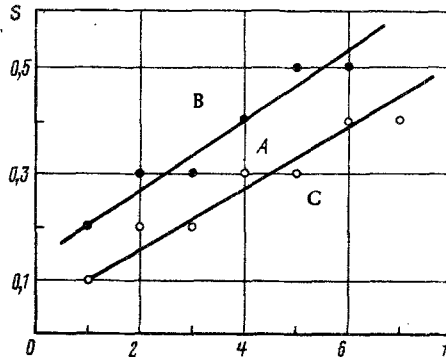


Fig. 1. Region of existence of active piston: A) active-piston region; B) gas-bubble fluidization region; C) immobile bed region;  $f$  in Hz.

and  $S/f = 0.1$  for the flow chopper. If  $f$  exceeds 1 Hz, the pressure waveform during one cycle reproduces all the elements obtained with  $f = 1$  Hz. The amplitude of the main pressure pulse is unaltered, and there is little change in the negative pressure arising under the bed, nor is there much variation in the amplitude of the secondary compression. This indicates that the bed behaves as a permeable piston for this range in  $f$ . The oscillograms serve to define the natural oscillation frequency of the bed. The recording for  $f = 1$  Hz indicates the half-period  $T/2$  for the natural oscillations as 0.1 sec. The recording for  $f = 5$  Hz shows that in this case the gas is supplied in resonance with the natural oscillation, and the most vigorous mixing occurs under these hydrodynamic conditions.

Therefore, it is necessary to predict or calculate the natural oscillation frequency of a bed of granular material in relation to the dimensions of the apparatus and the properties of the material. This is possible by mathematical description of the motion of the bed in response to the gas pressure pulse. We consider the motion of the gas in a space bounded at the top by a mobile gas-permeable bed of granular material and at the bottom by the base of the apparatus and the flow chopper. A mathematical description of this system (Fig. 3) includes the force-balance equation for the bed as a whole and equations for the gas mass balance in the layer under the bed and the state of the gas under isothermal conditions.

The following is the force-balance equation for the bed:

$$M \frac{d^2z}{d\tau^2} = -Mg + (P_1 - P_0)S_e - F_{fr}. \quad (1)$$

The lower boundary of the bed corresponds to the coordinate  $z_0$ , and the bed can move only in the region  $z > z_0$ ; the frictional force  $F_{fr}$  applied to the bed by the wall of the apparatus is determined by the method of [2]:

$$d\sigma_x S_e = \rho_{be}(1-\varepsilon)(g-a)(H-x)S_e - \rho_{be}(1-\varepsilon)(g+a)(H-x-dx)S_e + \tau_F dx \pi D. \quad (2)$$

This expression has been written for a cylindrical apparatus provided that the bed porosity  $\varepsilon$  remains constant; the tangential stress  $\varepsilon$  is related to the normal stress  $\sigma_x$  by

$$\sigma_x = \tau_F / \alpha \lambda, \quad (3)$$

where  $\lambda$  and  $\alpha$  are the lateral-pressure and friction coefficients.

Equation (2) can be integrated with the boundary condition  $\tau_F = 0$  for  $x = H$ :

$$\tau_F = \frac{A_1}{B} \{ \exp[B\alpha\lambda(H-x)] - 1 \}, \quad (4)$$

where  $A_1 = \rho_{be}(1-\varepsilon)(g+a)$ ;  $B = 4/D$ ; (1) contains the integral value of the frictional force acting on the entire surface of contact between the bed and the wall, so

$$F_{fr} = \int_0^H \tau_F dx \pi D = B_1(g+a)M, \quad (5)$$

where

$$B_1 = \frac{1}{4\alpha\lambda} \frac{H}{D} \left( \exp \cdot 4\alpha\lambda \frac{H}{D} - 4\alpha\lambda \frac{H}{D} - 1 \right).$$

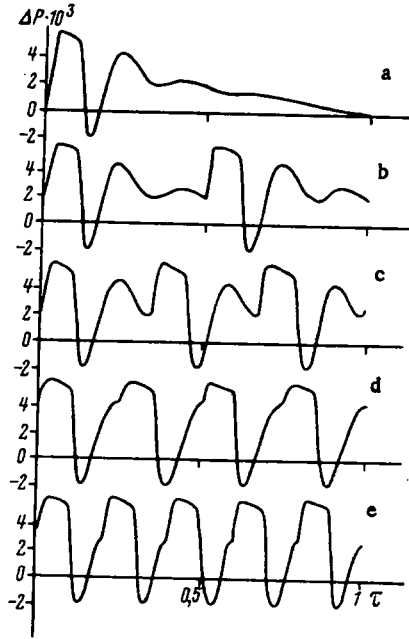


Fig. 2. Oscillograms for gas pressure under bed: a)  $f = 1$  Hz,  $S = 0.1$ ; b) 2 and 0.2, respectively; c) 3 and 0.3; d) 4 and 0.4; e) 5 and 0.5.

The frictional force is thus dependent on the instantaneous acceleration of the bed; we substitute the values into (1) to get

$$\frac{d^2z}{d\tau^2} = \frac{(P_1 - P_0) S_e}{M(1 + B_1)} - g. \quad (6)$$

The gas-balance equation for the space under the bed includes the rate of entry of the gas from the compressor and the rate of loss of gas through the bed. The bed may also be displaced, so we write

$$(G_1 - G_2) d\tau = d[\rho_1(S_e z + V_0)], \quad (7)$$

where  $G_1 = f(P_1)$  is the equation characterizing the compressor (in our case  $G_1 = [2-3 \cdot 10^{-4} (P_1 - P_0)] 10^{-2}$ ). The gas flow rate due to filtration through the lower boundary of the bed is given by

$$G_2 = A \left. \frac{\partial P}{\partial x} \right|_{x=0} S_e \rho_1, \quad (8)$$

and to use this we need to solve the equation for nonstationary passage of gas through a bed of granular material:

$$\frac{\partial P}{\partial \tau} = \frac{\partial}{\partial x} \left( \frac{c}{\mu \varepsilon} P \frac{\partial P}{\partial x} \right). \quad (9)$$

We assume that the porosity of the bed remains constant ( $\varepsilon = \text{const}$ ) and that the gas density varies only slightly within the bed ( $\rho = \text{const}$ ). These equations enable us [3] to transform the filtration equation to

$$\frac{\partial P}{\partial \tau} = A \frac{\partial^2 P}{\partial x^2}. \quad (10)$$

The filtration coefficient  $A$  is [2] given by

$$A = \frac{\varepsilon^3 \rho_0}{K(1 - \varepsilon)^2 a^2 \mu}, \quad (11)$$

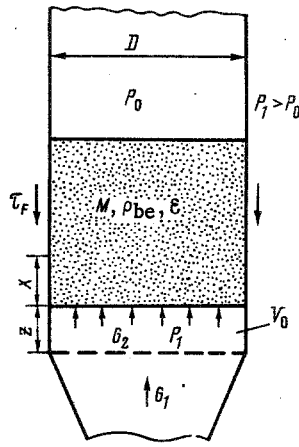


Fig. 3. Theoretical scheme for apparatus for purposes of model.

where  $a$  is the specific surface of the granular material in  $m^2/m^3$  and  $K$  is the Kozeny-Karman constant. These assumptions do not introduce much error into the equation describing the gas pressure under the bed, but they do not allow us to trace the propagation of the gas-compression wave within the bed, e. g., in the disruption of the latter.

The initial condition for solving (10) is characterized by the pressure distribution over the height of the bed  $P(x, 0) = \varphi(x)$ , while the boundary conditions that incorporate the variation in gas pressure under the bed are  $P(0, \tau) = P_1(\tau)$ ;  $P(H, \tau) = P_0$ .

The solution to (10) in analytical form is [4]

$$P(x, \tau) = P_1(\tau) - \frac{x}{H} [P_1(\tau) - P_0] + \sum_{n=1}^{\infty} \left[ \frac{2}{H} \int_0^H (\varphi(\eta) - P_1 + (P_1 - P_0) \times \frac{\eta}{H}) \sin \frac{\pi n}{H} \eta d\eta \right] \exp \left[ - \left( \frac{\pi n}{H} \right)^2 A^2 \tau \right] \sin \frac{\pi n}{H} x. \quad (12)$$

Equations (6) and (7) form a system that describes the pressure variation under the bed and the motion of the bed as a permeable piston:

$$\begin{aligned} \frac{d^2 z}{d\tau^2} &= \frac{P_1 - P_0}{M(1 + B_1)} S_e - g, \\ \frac{dP_1}{d\tau} &= - \frac{P_1 S_e}{S_e z + V_0} \frac{dz}{d\tau} + \frac{G_1 - G_2}{S_e z + V_0} \frac{P_0}{\rho_0}, \\ G_1 &= f(P_1). \end{aligned} \quad (13)$$

System (13) was solved by computer; the infiltration equation has to be solved at each step of integration in (13), while the boundary and initial conditions  $P_1(\tau)$  and  $\varphi(x)$  for defining  $P(x, \tau)$  in each successive step are generated from the result for the previous step.

Figure 4 shows the gas pressure variation under the bed during the cycle of the chopper operation as recorded and as calculated from (13); the calculated and observed curves are essentially the same up to point 1, but subsequently the calculated curve is only in qualitative agreement with experiment, although the period of the natural oscillation given by the calculated curve agrees satisfactorily with the observed period.

Figure 5 is a nomogram constructed for determining the natural frequency on the basis of the calculations from this model.

If  $f$  is derived from the mass of the bed, the volume of the space under the bed, and the diameter, one can determine the optimum conditions providing most rapid mixing of the granular material.

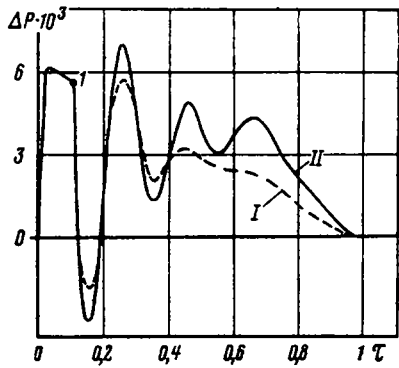


Fig. 4

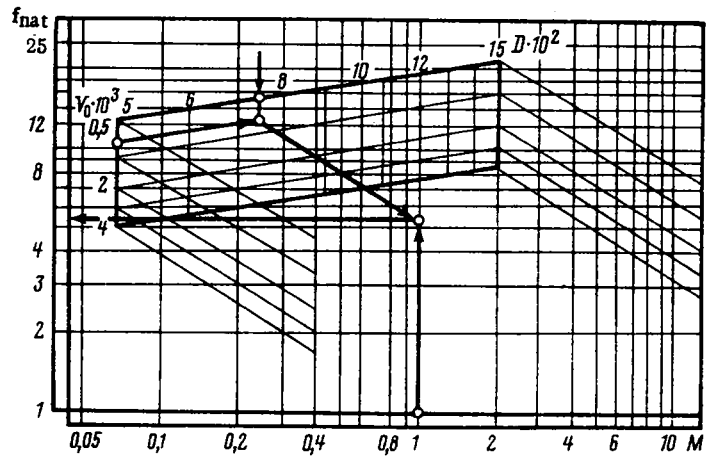


Fig. 5

Fig. 4. Variation in gas pressure under bed in one cycle: I) observed; II) theoretical curve;  $\Delta P = P_1 - P_0$ , Pa;  $\tau$ , sec.

Fig. 5. Nomogram for determining natural frequency of bed;  $D$  in m,  $V_0$  in  $m^3$ ,  $M$  in kg,  $f$  in Hz.

#### NOTATION

$A$	is the filtration coefficient, $m^2/sec$ ;
$D$	is the diameter; m;
$H$	is the depth of the bed, m;
$M$	is the mass of bed, kg;
$P_0, P_1$	are the gas pressures over the bed and under the bed, Pa;
$S_e$	is the grid area, $m^2$ ;
$V_0$	is the underbed volume, $m^3$ ;
$x$	is the ordinate in bed relative to lower boundary, m;
$z$	is the ordinate of the upper boundary relative to grid, m;
$\epsilon$	is the porosity;
$\mu$	is the dynamic viscosity, Pa · sec.

#### LITERATURE CITED

1. H. W. Wong and M. H. J. Baird, "Fluidization in a pulsed gas flow," Chem. Eng. J., 2, No. 2, 104-113 (1971).
2. M. É. Aéroov and O. M. Todes, Hydraulic and Thermal Working Principles of Equipment with Stationary and Fluidized Granular Beds [in Russian], Khimiya, Leningrad (1968).
3. V. I. Aravin and S. N. Numerov, Theory of the Motion of Liquids and Gases in an Underformable Porous Medium [in Russian], Gos. Izd. Tekh-Teor. Lit., Moscow (1953).
4. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1977).